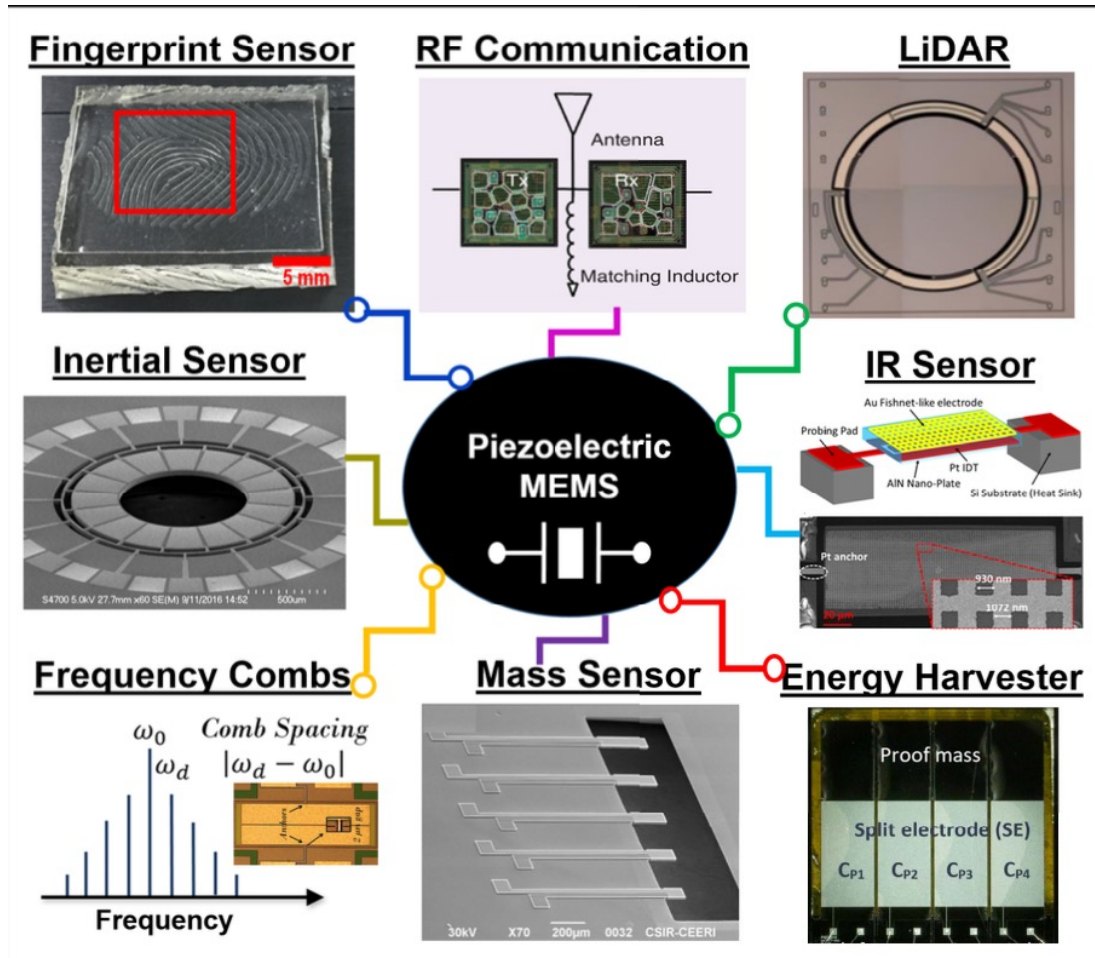


Piezoelectric scaling in MEMS

H. Shea

Piezoelectric MEMS



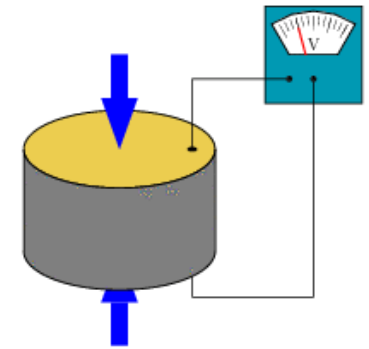
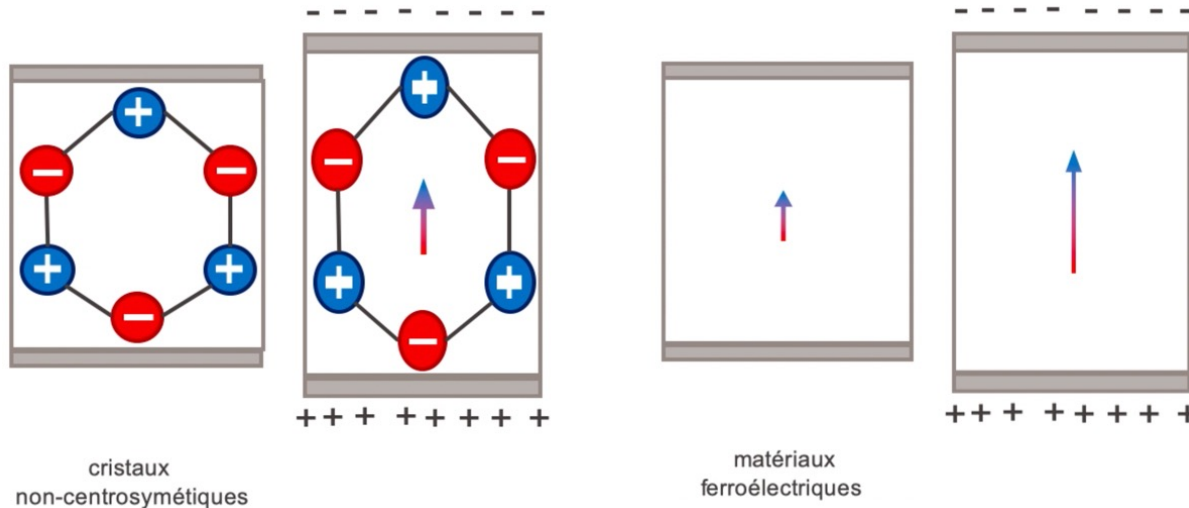
- Fast
- High Force
- Small Strain

G. Pillai and S.-S. Li, "Piezoelectric MEMS Resonators: A Review," *IEEE Sensors Journal*, vol. 21, pp. 12589–12605, 2021 doi: [10.1109/JSEN.2020.3039052](https://doi.org/10.1109/JSEN.2020.3039052)

- 1. Piezoelectric effect and main materials
- 2. MEMS-scale actuators
- 3. RF filters

1. Piezoelectric Effect

- Piezoelectricity
 - Material property through which:
 - An applied electric field leads to mechanical strain
 - Mechanical stress generates charges
 - It occurs in materials without inversion symmetry
 - Either the Unit lattice is not symmetric, or ferroelectric material



<https://en.wikipedia.org/wiki/Piezoelectricity>

Both ways! Direct and reverse

- Motion when apply a voltage
- Generate voltage/current when impose motion

Piezoelectric Actuation

- Examples of piezo materials:

- **PZT**: $\text{Pb}(\text{Zr,Ti})\text{O}_3$
- Quartz
- AlN
- **AlScN** AlScN (depends on Sc content)
- ZnO
- LiNbO_3
- **PMN-PT**: $0.7 \text{Pb}(\text{Mg}_{1/3}\text{Nb}_{2/3})\text{O}_3 - 0.3 \text{PbTiO}_3$
- **PVDF** Poly(1,1-difluoroethylene)

Black: crystal non-centrosymmetric
Red: ferroelectric

- Fast (up to GHz)
- Linear in strain-voltage and stress-voltage
- High efficiency
- High Young's modulus (of order 60 GPa except for polymers)
- Small displacements (strain typically 0.1%)
- Highest actuator performance materials (PZT) contain Pb (Lead). Challenging to find good Pb-free options for actuation.

Piezoelectric effect: links deformation (i.e. strain, hence stress) and electric polarization

- An applied Electric field leads to mechanical strain
- Mechanical stress generates charges

$$\epsilon_{ij} = S_{ijkl} \sigma_{kl} + d_{ijk} E_k$$

$$D_i = \epsilon_{ij} E_j + d_{ijk} \sigma_{jk}$$

Purple: all materials

Green: only piezo

S_{ijkl} Compliance tensor (inverse of Elasticity tensor)

ϵ_{ij} Strain (if single index: 1,2,3: axial, 4,5,6: shear)

σ_{kl} Stress (if single index: 1,2,3: axial, 4,5,6: shear)

ϵ_{ij} Permittivity matrix

E_k Electric field components (not Young's modulus!)

D_i Displacement field (electric)

d_{ijk} Piezo tensor

$$\vec{D} = \epsilon \vec{E} + \vec{P}$$

\vec{P} Polarization: density of electric dipoles.
units: C/m²

Piezo matrix links charge and stress (or voltage and strain)

The displacement field D is equivalent to a surface charge, C/m^2

The surface charge density is equal to the remanent internal polarization

$$D_i = d_{ijk} \sigma_{kl}$$

The units of the piezoelectric coefficients d_{ij} is C/N (i.e. Coulomb per Newton), or m/V (i.e. Meter per Volt)

$$\begin{pmatrix} q_x \\ q_y \\ q_z \end{pmatrix} = \begin{pmatrix} d_{11} & d_{12} & \dots & \dots & \dots & d_{16} \\ d_{12} & \dots & \dots & \dots & \dots & \dots \\ \underbrace{d_{31} & d_{32} & \dots}_{axial} & \dots & \dots & \underbrace{\dots & \dots & d_{36}}_{shear} \end{pmatrix} \cdot \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_4 \\ \tau_5 \\ \tau_6 \end{pmatrix}$$

Charge per unit area

stress

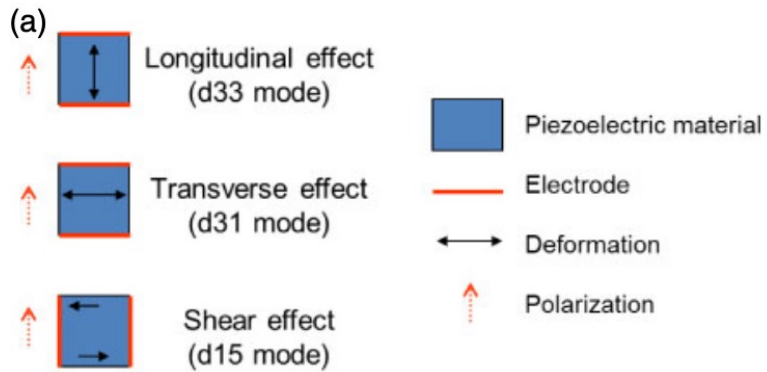
$$\begin{pmatrix} \varepsilon_x \end{pmatrix} = \begin{pmatrix} d_{11} & \dots & \dots \\ d_{31} & \dots & \dots \end{pmatrix}^t \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

The piezo matrix has 18 components (6x3)

- The left side of the matrix represents piezoelectric coefficients linked to **axial** deformations
- The right side of the matrix represents piezoelectric coefficients linked to **shear** deformations

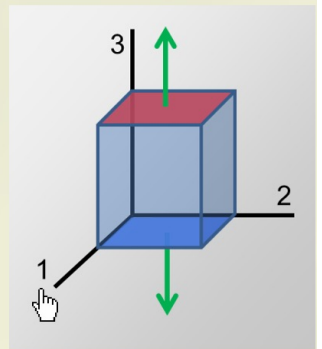
$$d_{ik}$$

k = direction of mechanical action
 i = direction of the electric field
 = direction perpendicular to the electrodes



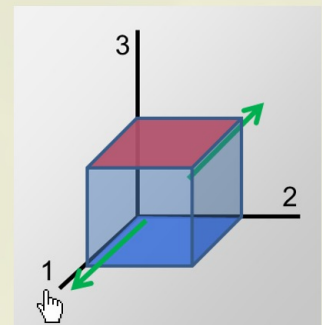
Compression Mode

d_{33} would indicate the polarization generated in the 3-direction per unit of mechanical compression stress (T) applied in the 3-direction to the piezoelectric body.



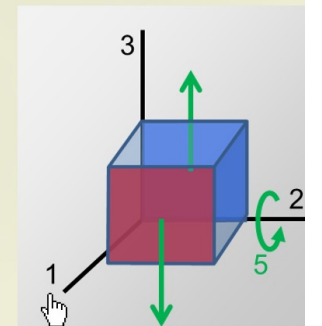
Transverse Mode

d_{31} is the polarization developed in the 3-direction per unit stress applied in the 1-direction (all other external stresses = 0).



Shear Mode

d_{15} means the polarization developed in the 1-direction per unit shear stress 5 applied (= shear around the 2-direction) when there are no other external stresses.



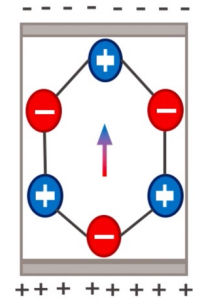
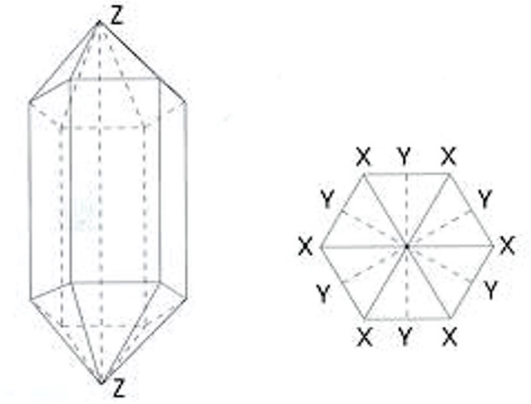
Piezo matrix for quartz

$$\begin{pmatrix} q_x \\ q_y \\ q_z \end{pmatrix} = \begin{pmatrix} d_{11} & -d_{11} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{pmatrix} + \begin{pmatrix} d_{14} & 0 & 0 \\ 0 & -d_{14} & -2d_{11} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{pmatrix}$$

Five non-zero coefficients and only 2 values (due to symmetry)

$$d_{11} = 2.3 \cdot 10^{-12} \text{ C/N}$$

$$d_{14} = -0.7 \cdot 10^{-12} \text{ C/N}$$



- When using a piezo element as a sensor, C/N (charge per applied force) is more convenient (usually pC/N)
- When using a piezo as an actuator, m/V is more intuitive (usually pm/V)
- C/N = m/V

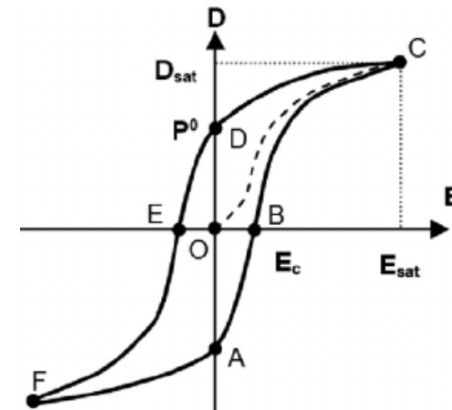
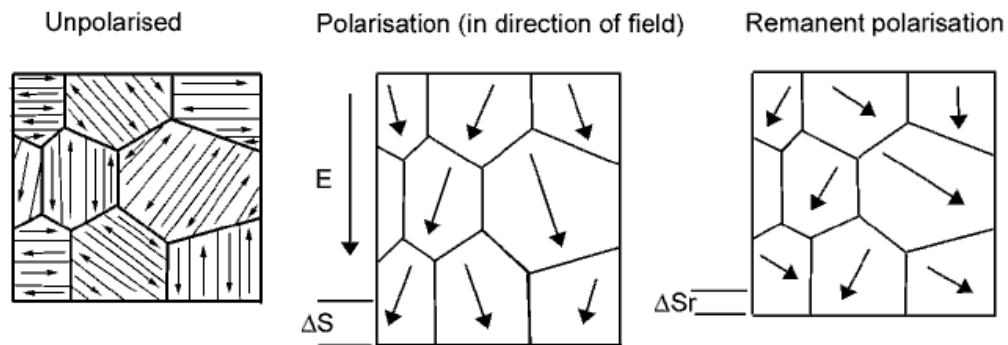
But Quartz is not easily MEMS compatible...

Ferroelectric materials

In a ferroelectric material, the dipoles are coupled

The coupling of the dipoles makes them line up with each other, creating ferroelectric domains. One can impose polarization by applying an external field and heating.

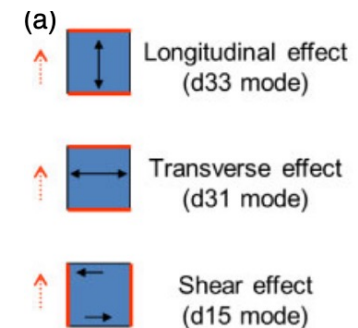
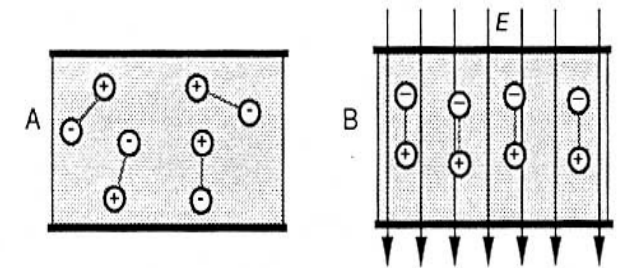
Curie Temperature T_C : above which the spontaneous polarization disappears. Need to stay below T_C unless wish to remove the polarization



Piezo Matrix for piezo-ceramics

- Ferroelectric ceramics must be polarized (poled) to become piezoelectric.
- Polarization is obtained by applying a high electric field (typically just below breakdown) at high temperature, typically the final fabrication step.
- The Polarization direction is termed the z axis (by convention)

$$\begin{pmatrix} q_x \\ q_y \\ q_z \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix} \cdot (\sigma_{1...6})$$



Five non-zero coefficients and only 3 different values

For PZT:

$$d_{33} = 375 \cdot 10^{-12} \text{ C/N} \quad d_{31} = -190 \cdot 10^{-12} \text{ C/N} \quad d_{15} = 515 \cdot 10^{-12} \text{ C/N}$$

The relative permittivity of ceramic piezoelectric materials is very high: $\epsilon_r = 1200$

Comparing Piezo materials

1. Non-ferroelectric materials: Quartz (T.=570 °C)

Advantages

- Mechanical robustness
- Linearity, no hysteresis
- Can be used up to 400° C (transition temperature 573° C)
- No pyroelectric effect (no T dep)
- No aging

Drawbacks

- Price (~10'000 x more expensive than PZT)
- Requires a charge amp (weak couplings)
- Difficult to assemble

- good for precision devices like oscillators.
- But can't integrate Quartz in MEMS flow (A/N however OK)

2. Ferroelectric materials: PZT ceramics (Lead - Zirconium - Titanium, + oxides), BaTiO, Single crystals: LiNbO, (T=1210 °C),...

Advantages

- low cost
- High piezoelectric modules
- High electrical permittivity (1200) → High capacitance
- Machining/casting possible
- Adjustable Curie temperature (depending on composition)

Drawbacks

- Thermal dependence of coefficients
- Thermal hysteresis

- good for applications requiring high energy density.
- Can bond thin films to wafer

3. Polymer piezo materials

Polyvinylidene Fluoride (PVDF) is the most widely used piezoelectric polymer. The piezoelectric effect in PVDF arises from its molecular structure. $d_{33} = -33$ pC/N.

Advantages:

- **Flexibility and Processability:** can be manufactured into large areas, thin sheets and various shapes, offering design flexibility.
- **High Piezoelectric Stress Constants:** PVDF has higher piezoelectric stress constants (g_{33} , units of V. m /N) than ceramics like PZT, ie better sensitivity in sensor applications.

$$g_{33} = \frac{d_{33}}{\epsilon_0 \epsilon_r}$$

- Lightweight, flexible
- Can be cast as thin film

Drawbacks:

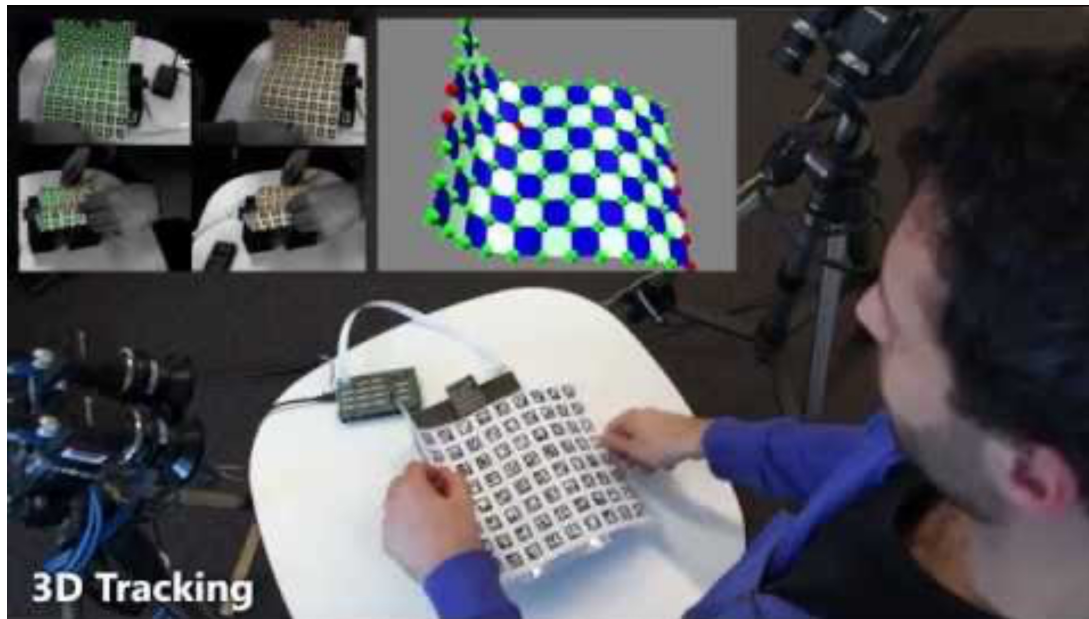
- Pyroelectric effect (leads to thermal drift)
- Low Curie temperature (120 °C)
- Mechanical damping
- Lower Electro-Mechanical Coupling Factor

Property	PVDF (Polyvinylidene Fluoride)	PZT (Lead Zirconate Titanate)	Quartz	BaTiO ₃ (Barium Titanate)
Density (g/cm ³)	1.78-1.80	7.5-8.0	2.65	~6.0
Dielectric Constant	10-12	300-1200	~4.5	1200-1700
d_{33} (pC/N)	-20 to -33	200-600	~2.3	~190-350
Pyroelectric Constant ($\mu\text{C}/\text{m}^2\text{K}$)	~30	~300	~0.1	~100
Elastic Modulus (GPa)	2-4	50-70	~80	~30-35
Electromechanical Coupling Constant (k^2)	~0.12	0.35-0.7	~0.1	~0.45-0.5

Source: ChatGPT, 12-2024

EPFL Flexible PVDF-based sensors

- PVDF-TrFE
 - Can be printed
 - Then needs to be poled using a very high electric field



<https://www.pyzoflex.com/>

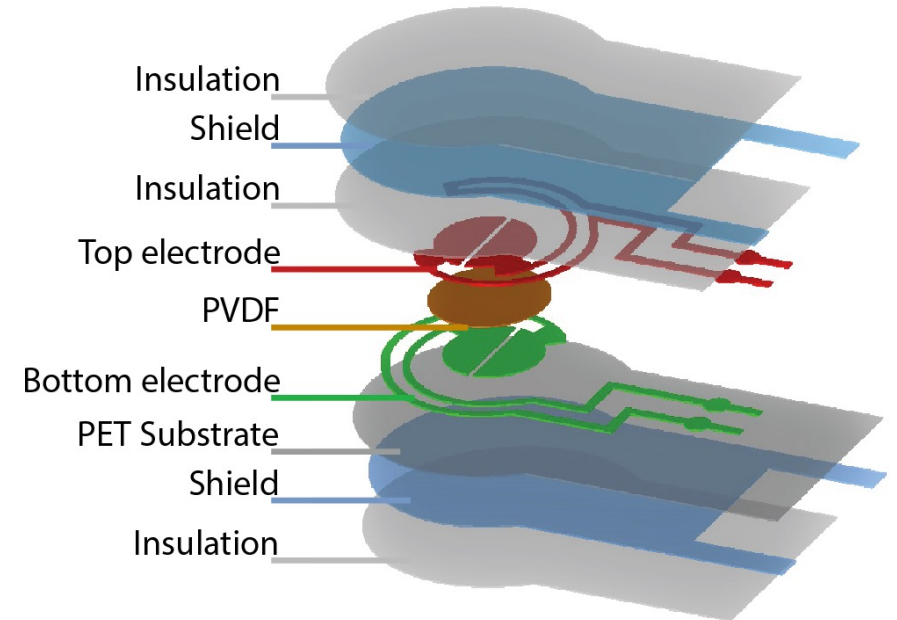
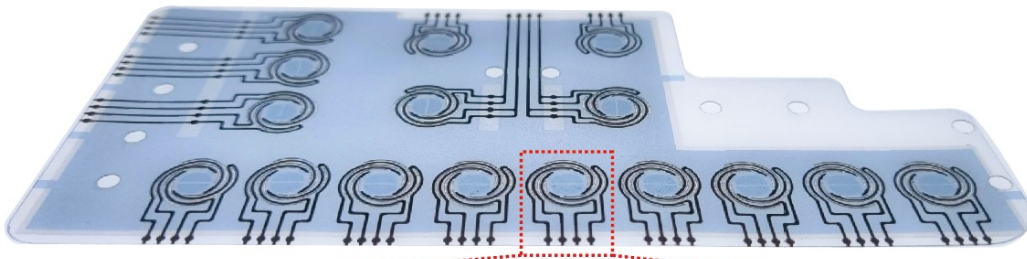
Product name	Typical TrFE content (mol%)	Curie Temperature (°C)	Typical remnant polarization at 150 V/ μm (mC/m ²)	Typical d_{33} (pC/N)
Piezotech® FC20	20	136	85	-24 to -30
Piezotech® FC25	25	115	70	-24 to -30
Piezotech® FC30	30	100	65	-18 to -22
Piezotech® FC45	45	60	45	-18 to -22

<https://piezotech.arkema.com/en/Products/piezoelectric-copolymers/>

EPFL Sensing example from EPFL-LMTS with PyzoFlex layer from Joanneum

At LMTS, we integrated (and shielded) piezoelectric PyzoFlex sensors from partner Joanneum Research (Austria) on top of the HAXELs enabling:

- Proximity sensing (finger approach, thermal)
- Force sensing (finger pushes, mechanical)



EPFL

Sensorized PopTouch reconfigurable user interface, with intuitive haptic feedback, on surfaces such as back of a smart phone.



Video – real
time

Buttons follow the slider.... but the slider can also follow button



Video – real time

EPFL Piezoelectric Actuation (linear)

- An electric field leads to mechanical strain, and to motion of amplitude z
- Direct expansion $\rightarrow z = d_{33}V$

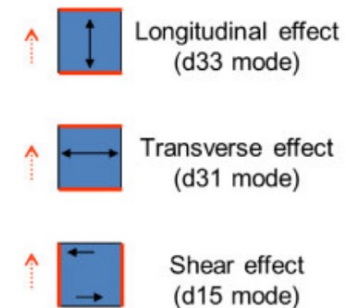
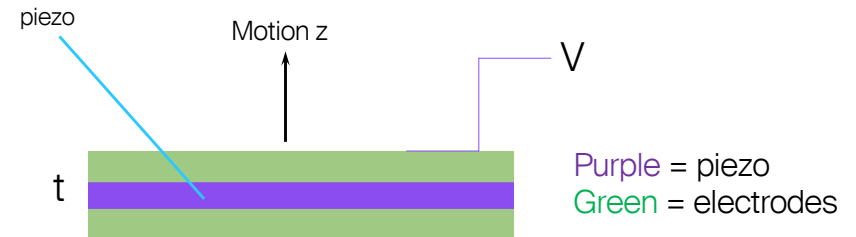
Derivation from strain

$$\epsilon_z = d_{33}E_z = \frac{d_{33}V}{t}$$

$$z = \epsilon_z t = d_{33}V$$

- d_{33} in $\left[\frac{pm}{V}\right]$, $AlN \rightarrow \frac{6pm}{V}$, $PZT \rightarrow 100 \text{ to } 200 \frac{pm}{V}$
- z -motion has No Dependence on film Thickness !
 - actuation displacement and force scale with V , not V/t
 - different from capacitive actuation (scales as E^2)

- z scales as L^0 (if assume V scale invariant).
- z scales as L^1 if assume V limited by breakdown



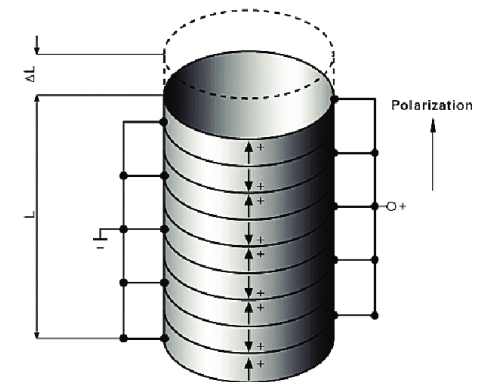
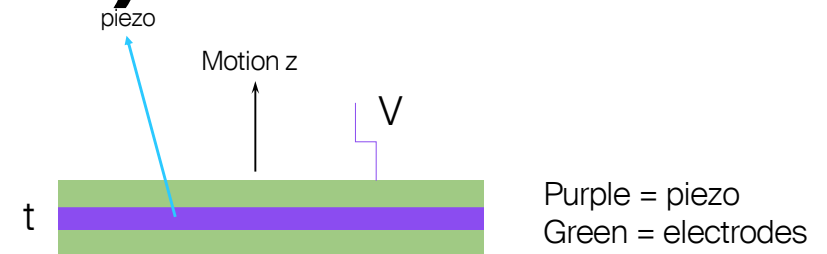
Piezoelectric Actuation (linear)

- Voltage is limited by breakdown voltage (or by power supply). Often up to 200 V
- Output force links stress and strain via Y (Young's modulus).
 - $F = P A = A Y \cdot \epsilon_z = A Y \frac{d_{33} V}{t} = A Y d_{33} E$
- F scales as L^2
- To get higher displacement: use multilayers. This is very common in cm-scale devices

- Multilayer: $z = d_{33} V * N$

$$\text{Energy density} = \frac{1}{2} \epsilon^2 = \frac{1}{2} \left[\frac{d_{33} V}{t} \right]^2 \propto L^{-2} \text{ or } L^0$$

Stacks are rarely used in MEMS.

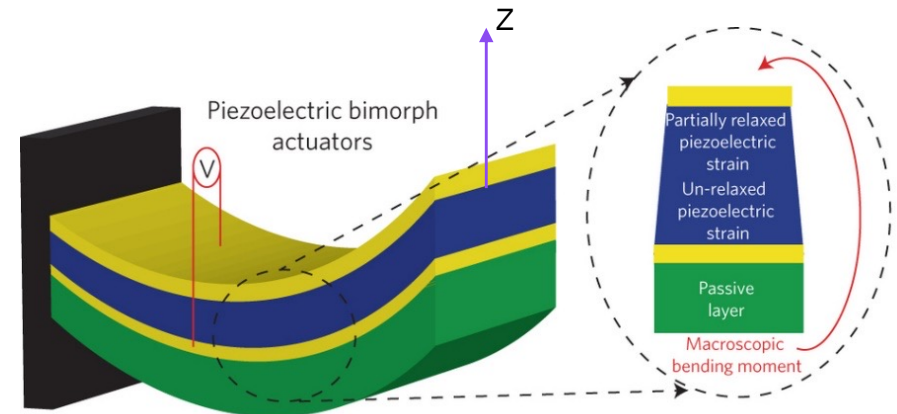
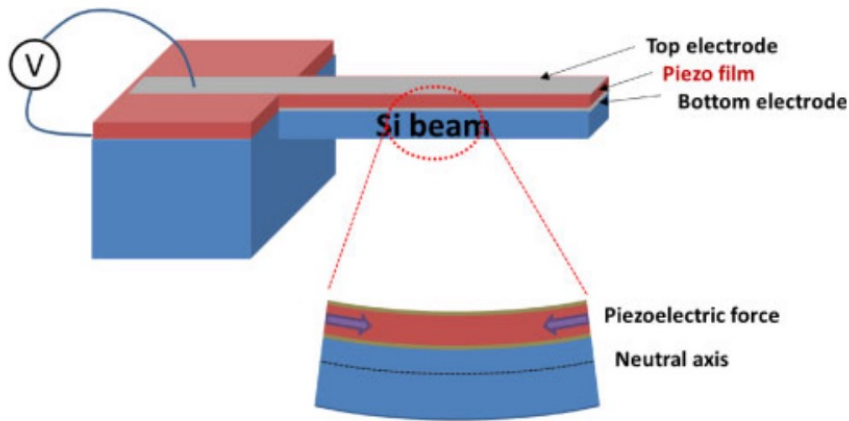
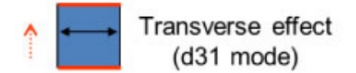


https://www.researchgate.net/figure/Structure-of-a-PZT-stack3_fig3_271882331



<https://piezodirect.com/multilayer-piezo-actuators/>

Piezoelectric Actuation (bending)



Kanno, I. Piezoelectric MEMS: Ferroelectric thin films for MEMS applications. *Jpn. J. Appl. Phys.* **57**, 040101 (2018).
<https://iopscience.iop.org/article/10.7567/JJAP57.040101>

- Make bending bimorph: piezo layer expands, the beam bends

- Coupling through bending moment $\rightarrow z = \chi_A \frac{z_{offset}}{t} \left(\frac{L}{t}\right)^2 d_{31}V = \chi_A z_{offset} \frac{L^2}{t^3} d_{31}V$

Assumes: all layers have the same Young's modulus (so E disappears from equations)

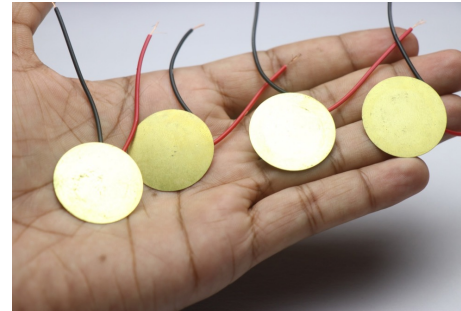
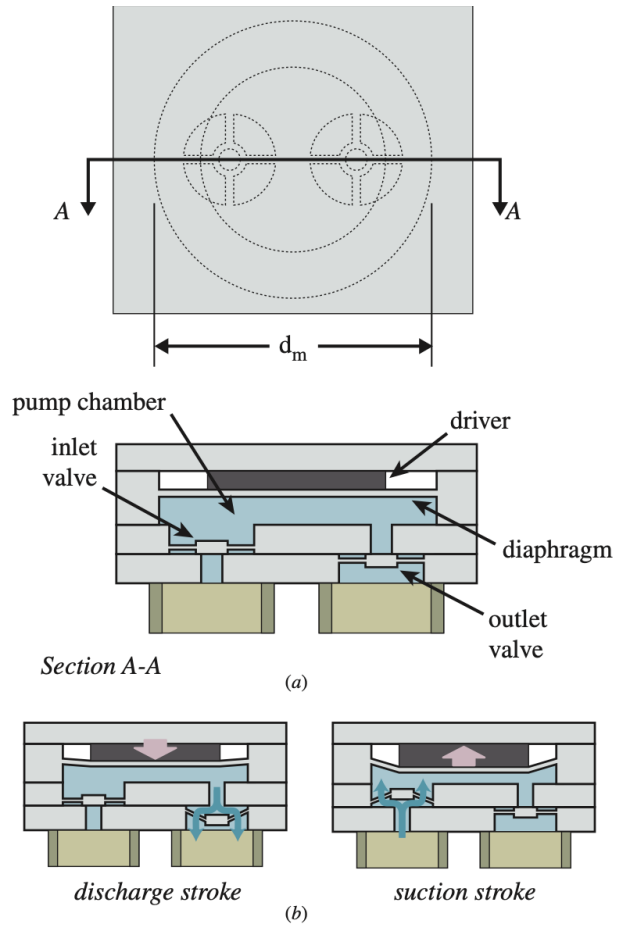
z_{offset} : distance from beam neutral axis to center of piezo films

Z Scales as L^0 or
as L^1 (if V scales as L)

2. Piezo-actuation in MEMS

- **Micropumps and microvalves** – for fluid control in lab-on-a-chip and biomedical devices.
- **Micromirrors and optical scanners** – for beam steering in optical MEMS and LIDAR.
- **Precision positioning** – in AFM probes and nanopositioning stages.
- **Ultrasonic transducers** – PMUTs for imaging (e.g., in medical ultrasound) or sensing.
- **Energy harvesting** – converting vibrations into electrical energy in self-powered MEMS.

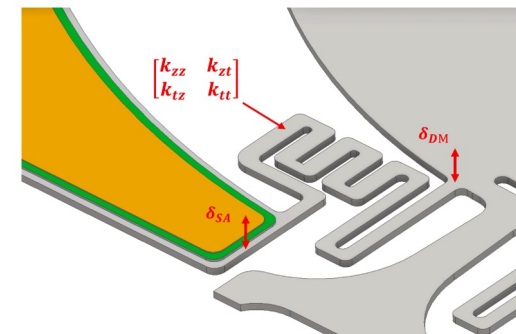
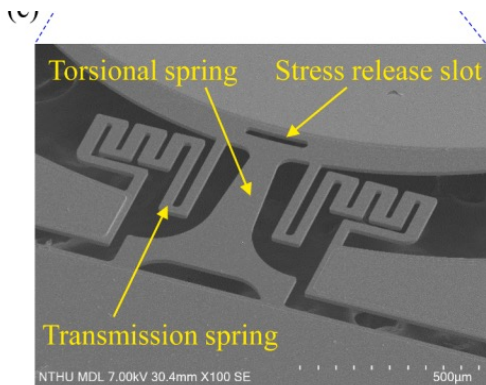
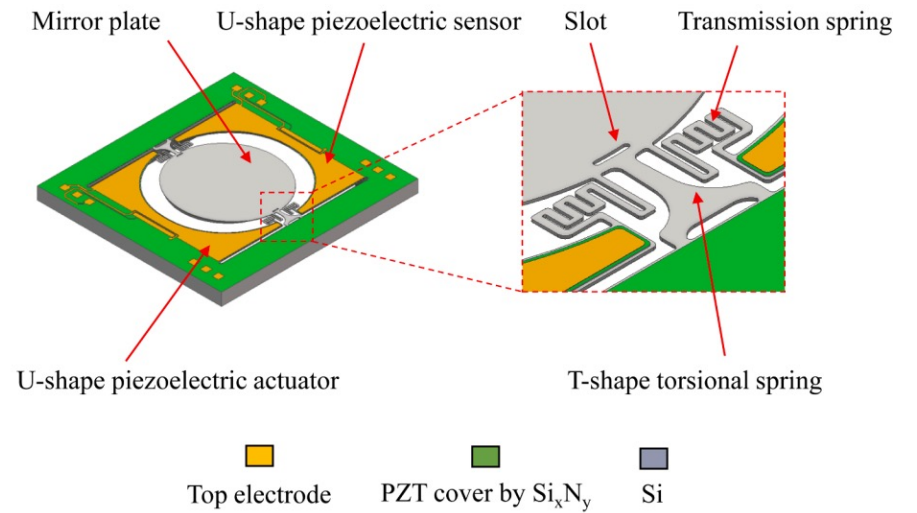
Micro-pump



D. J. Laser, J. G. Santiago, A review of micropumps. *Journal of Micromechanics and Microengineering* 14, R35-R64 (2004).

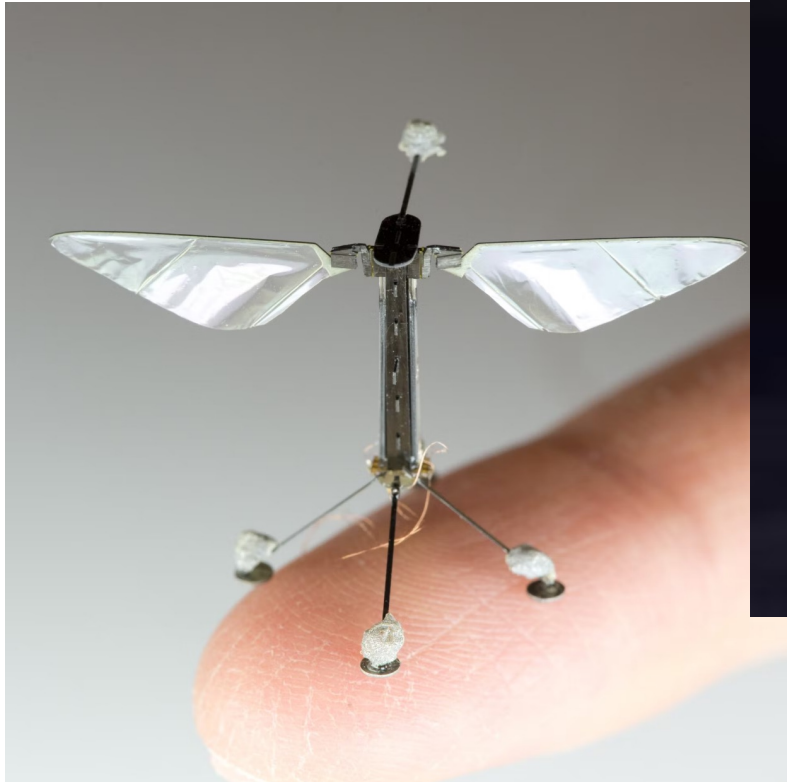
Scanning mirror

- Need transmission to convert small strain to large motion
- Can operate at kHz
- Was for initially projectors, now for LIDAR



H.-C. Cheng, et al, On the design of piezoelectric MEMS scanning mirror for large reflection area and wide scan angle. *Sensors and Actuators A: Physical* **349**, 114010 (2023).

EPFL



<https://vimeo.com/65313515>

Rob Wood and co, Robobee, Harvard University

Piezo bimorph (2 antagonistic), 200 Hz

K. Y. Ma, P. Chirarattananon, S. B. Fuller, and R. J. Wood, "Controlled Flight of a Biologically Inspired, Insect-Scale Robot," *Science*, vol. 340, no. 6132, pp. 603–607, May 2013, doi: [10.1126/science.1231806](https://doi.org/10.1126/science.1231806).

Piezomotor, mm-scale (or less)

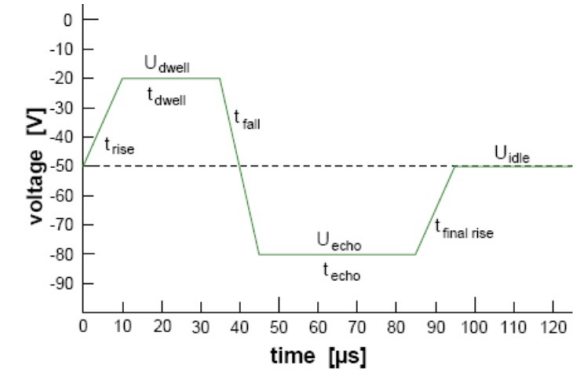
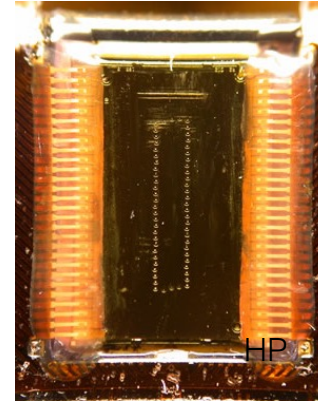
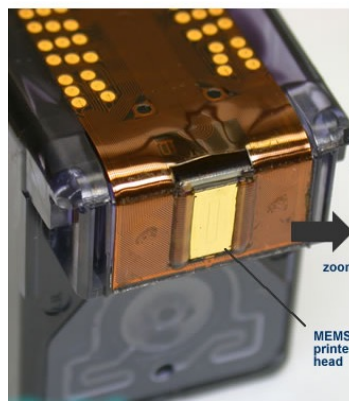


<https://www.youtube.com/watch?v=7iHL4ZCkCKc>

nm steps at many kHz

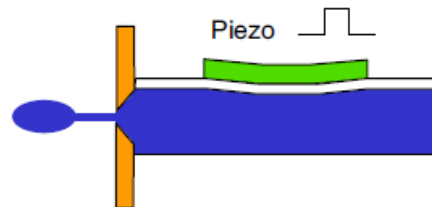
<https://www.piezomotor.com/>

Piezoelectric Actuation at mm scale, μm motion: inkjet printing



Inkjet printheads - Drop on Demand (DoD)
http://microfab.com/index.php?option=com_content&view=article&id=93&Itemid=9

Piezo



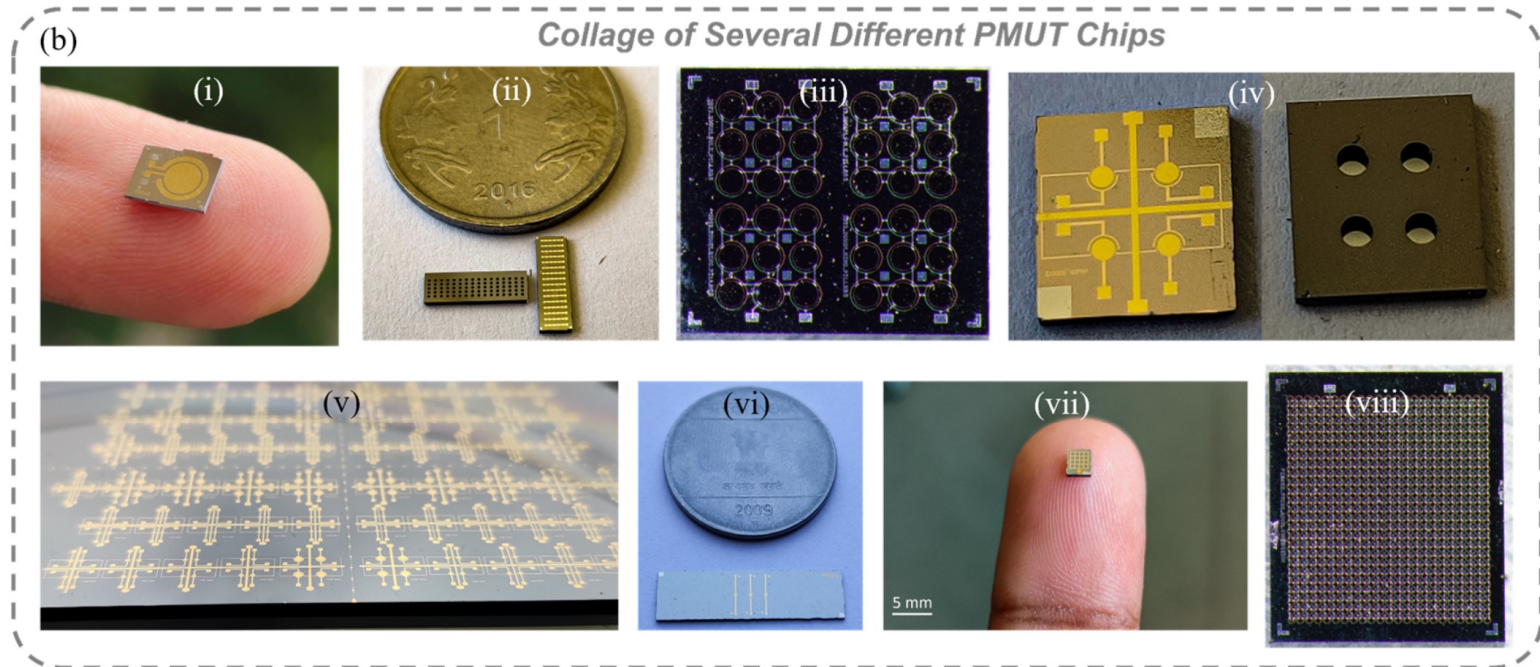
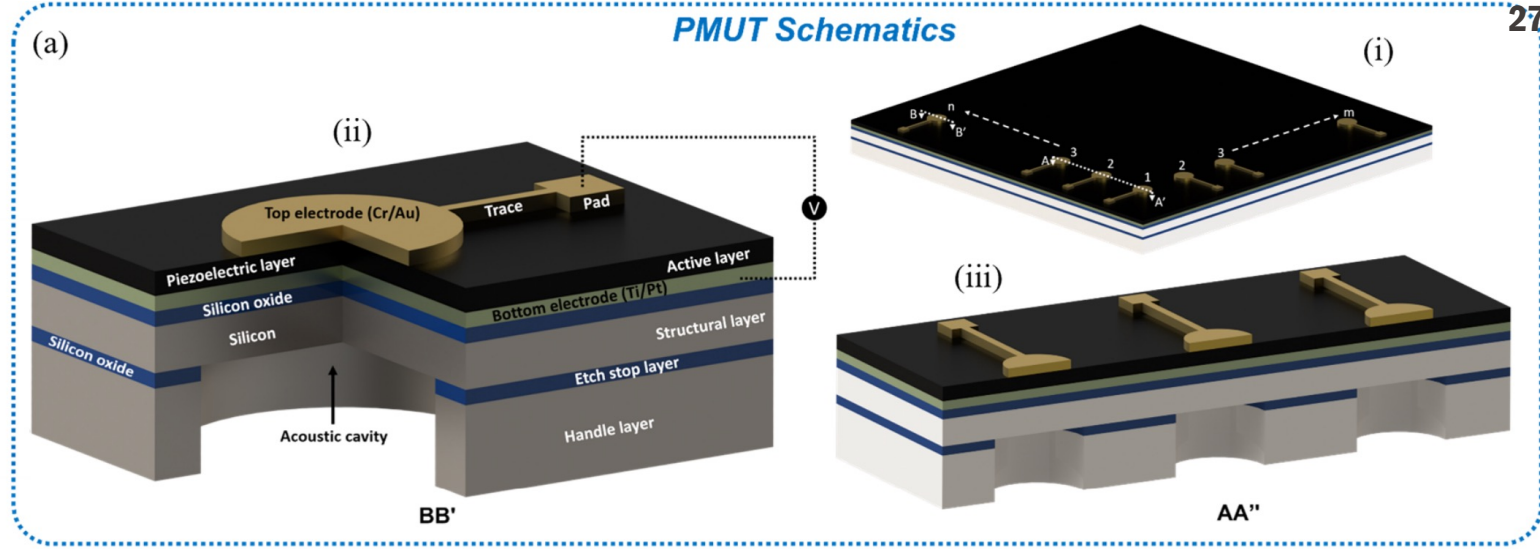
■ Piezo crystal deforms when electrical pulse applied – many different architectures

Operation at >10 kHz

EPFL **PMUT:**
Piezoelectric
Micromachined
Ultrasound
Transducers

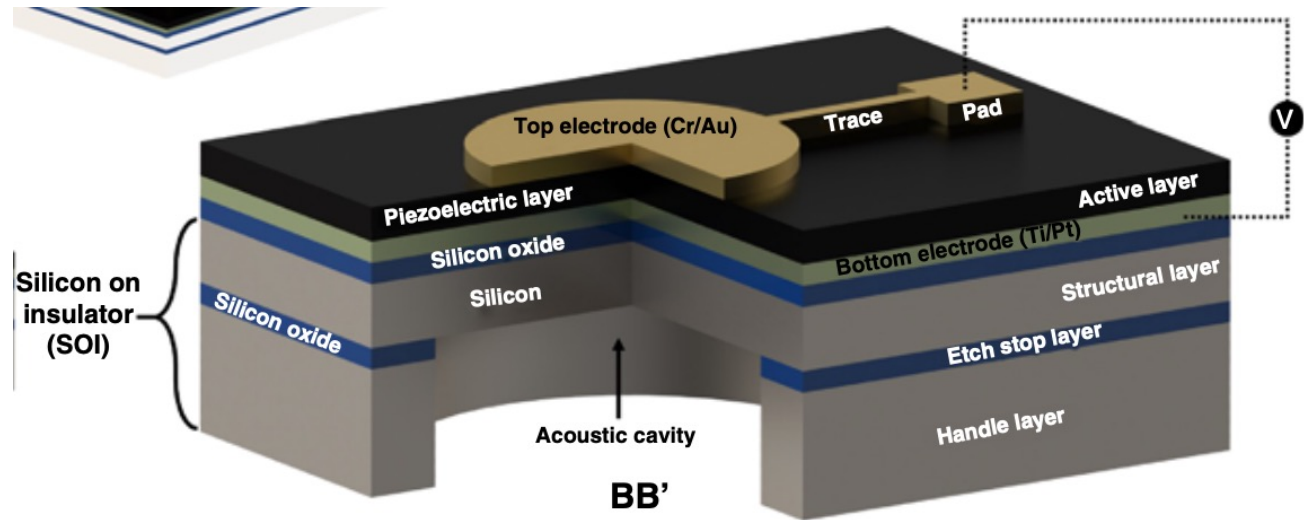
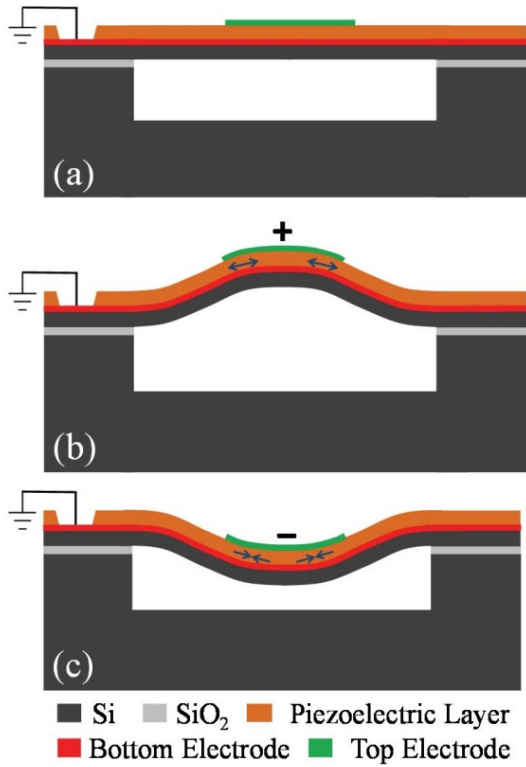
K. Roy, J. E.-Y. Lee, C. Lee, Thin-film PMUTs: a review of over 40 years of research. *Microsyst Nanoeng* 9, 1–17 (2023).

<https://www.nature.com/articles/s41378-023-00555-7>



EPFL Ultrasound sources (PMUT)

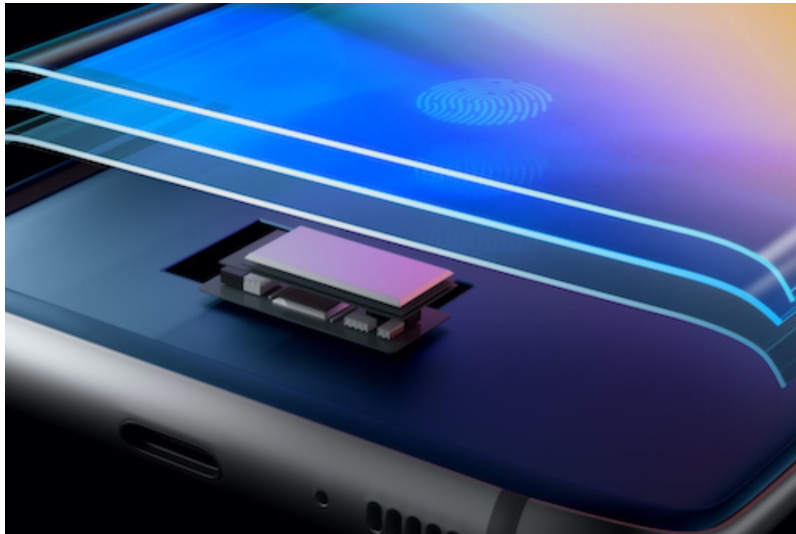
piezoelectric micromachined ultrasonic transducer. μm -scale



Roy et al, Microsystems and Nanoengineering 2023
doi: 10.1038/s41378-023-00555-7.

Y. Lu and D. A. Horsley, JMEMS 2015,
doi: [10.1109/JMEMS.2014.2387154](https://doi.org/10.1109/JMEMS.2014.2387154).

PMUT: To both sense and actuate

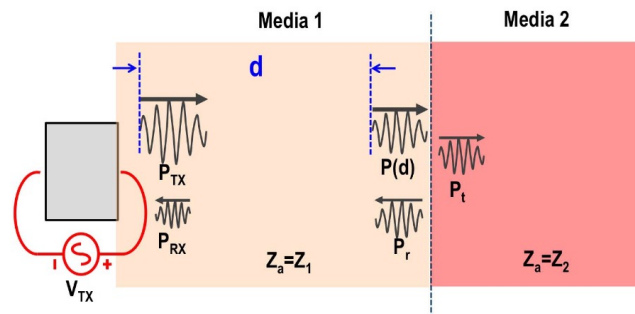


Qualcomm chip in Samsung phone
Under display

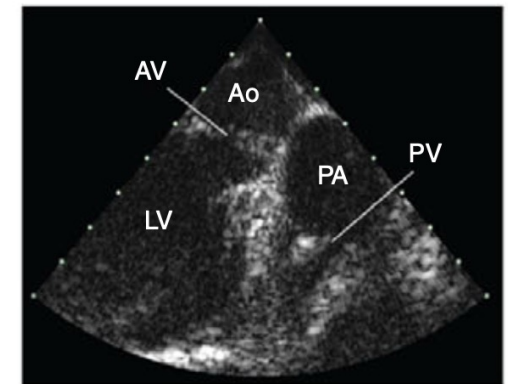
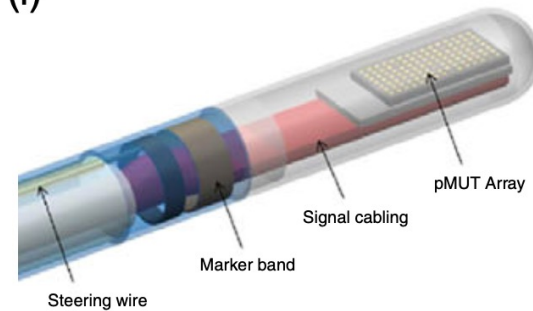
	Optical (TIR)	Capacitive	Ultrasound	
			Epidermal	Dermal
Dry Finger				
Wet Finger & Dried				
Wet Finger				

<https://www.nature.com/articles/s41378-023-00555-7>

Clinical & industrial imaging

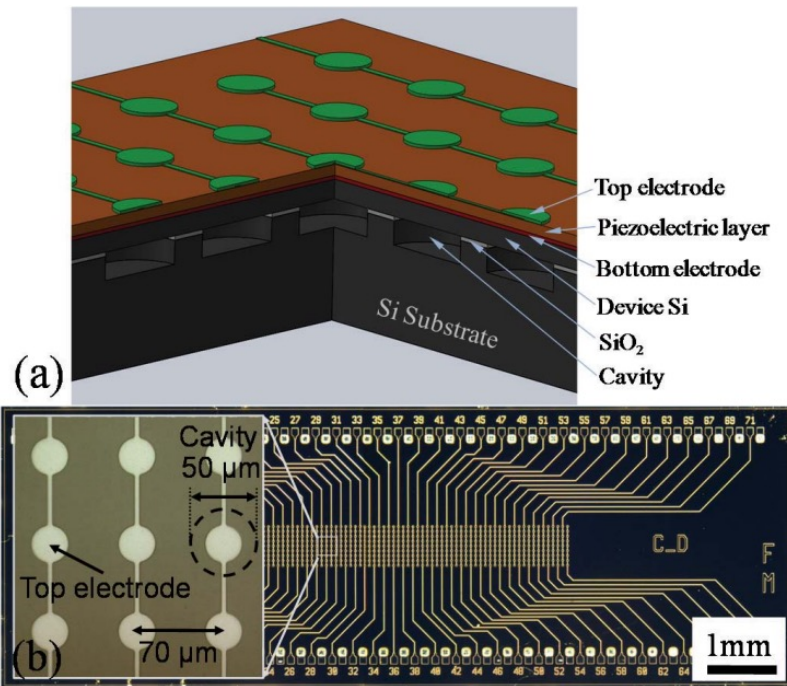


(i)



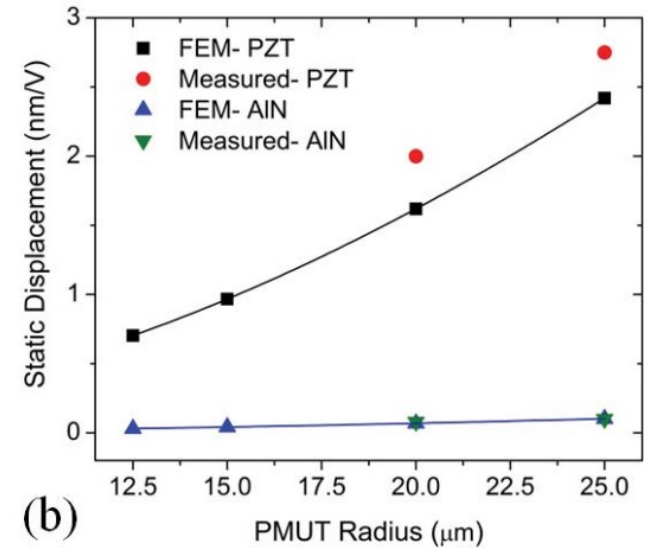
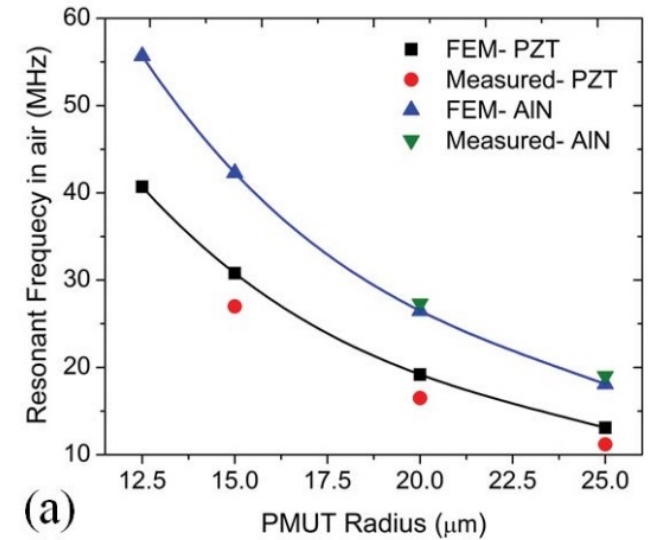
EPFL Ultrasound sources (PMUT)

piezoelectric micromachined ultrasonic transducer. μm -scale



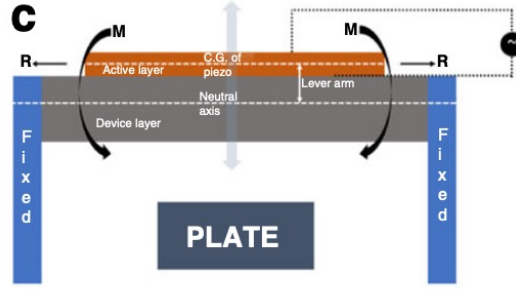
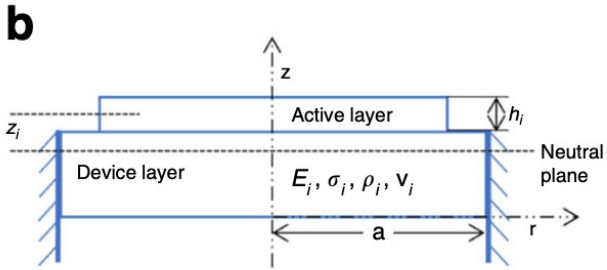
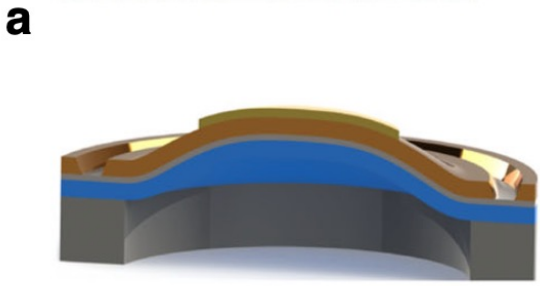
PMUT: Must sense and actuate

13-55 MHz



Scales up to loudspeakers

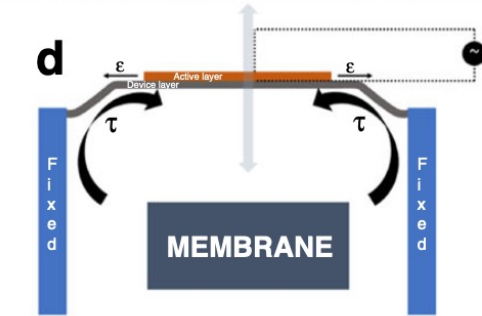
PMUT structural classification



Flexural rigidity (D_e) dominated

$$D_e = \sum_i \left(\frac{E_i}{1 - \nu_i^2} \right) \left(\frac{h_i^3}{12} + z_i^2 h_i \right)$$

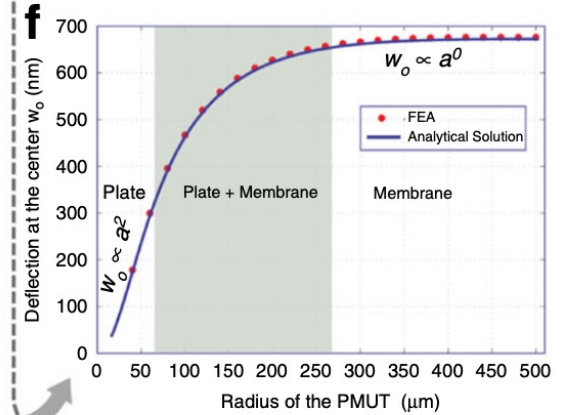
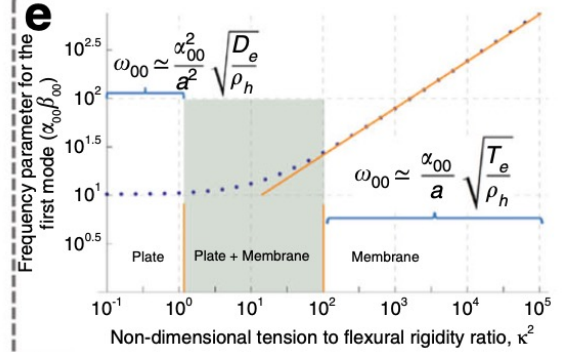
E_i – Young's Modulus of the i^{th} layer
 ν_i – Poisson's ratio of the i^{th} layer
 h_i – Thickness of i^{th} layer
 z_i – Distance of the i^{th} layer from the neutral axis



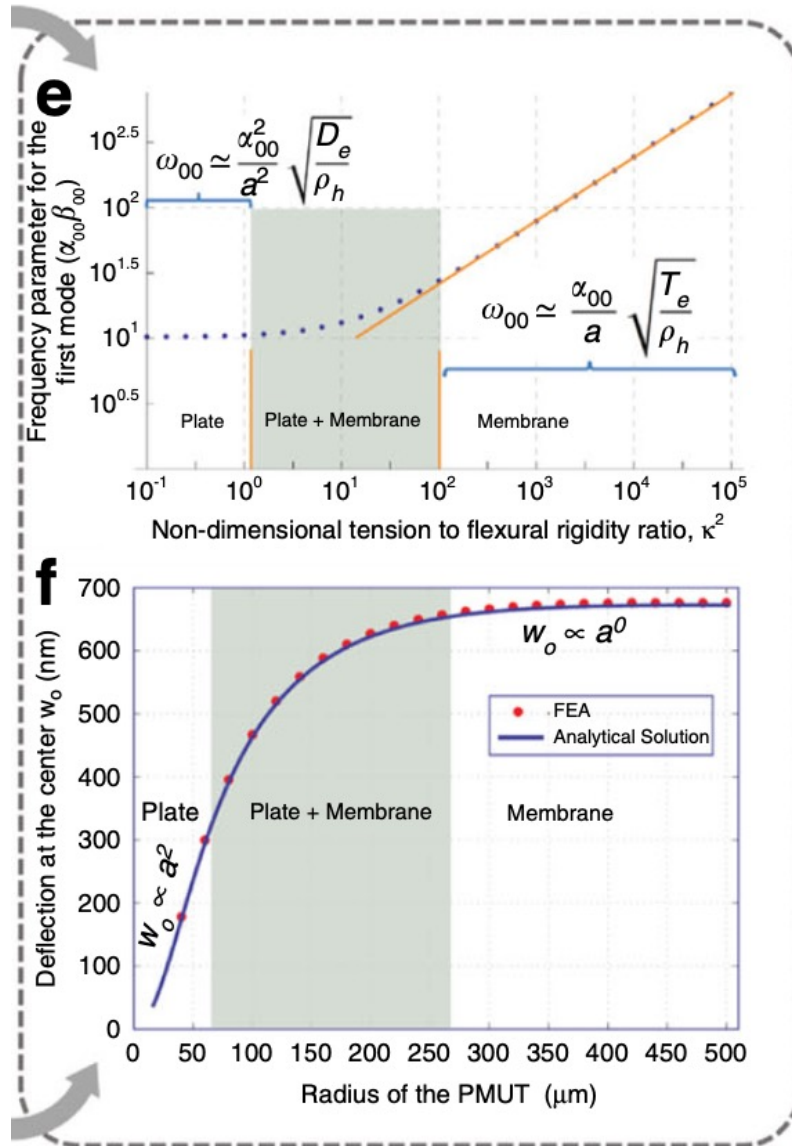
Pretension (T_e) dominated

$$T_e = \sum \sigma_i h_i$$

σ_i – Prestress of the i^{th} layer

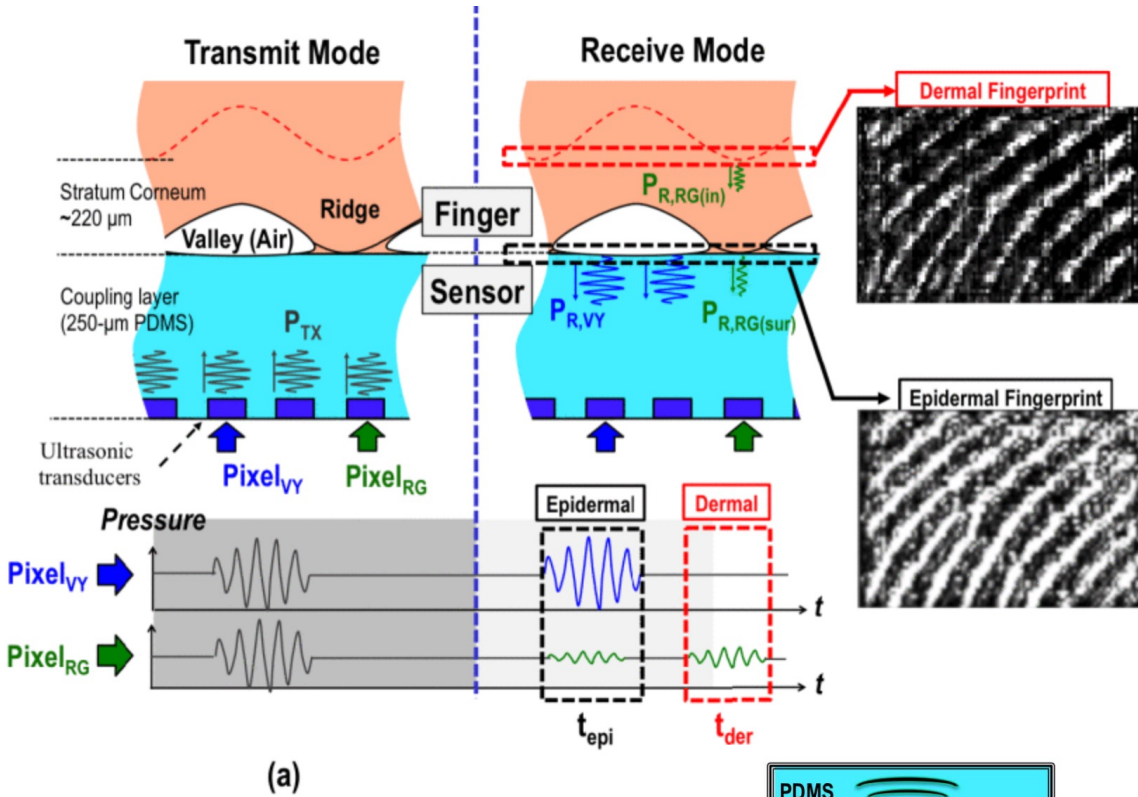


Roy et al, Microsystems and Nanoengineering 2023
 doi: 10.1038/s41378-023-00555-7.



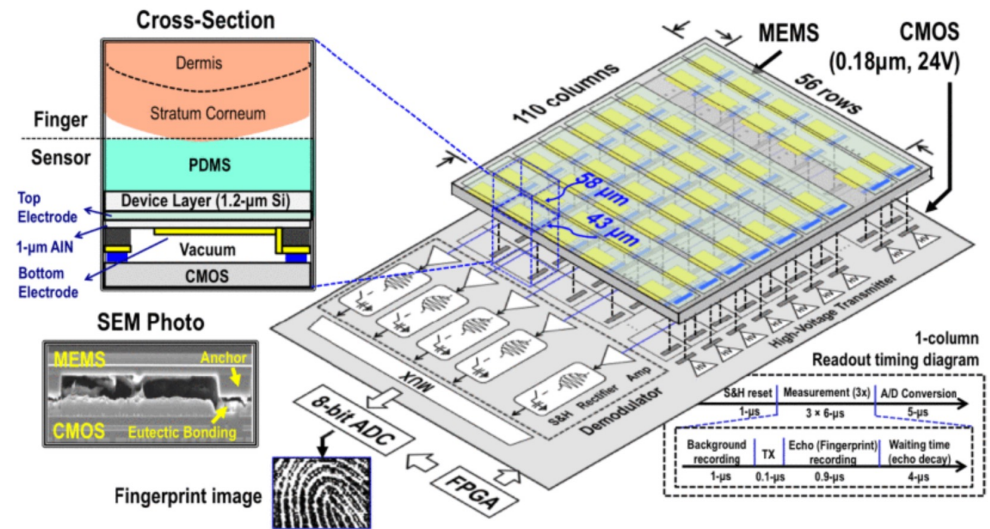
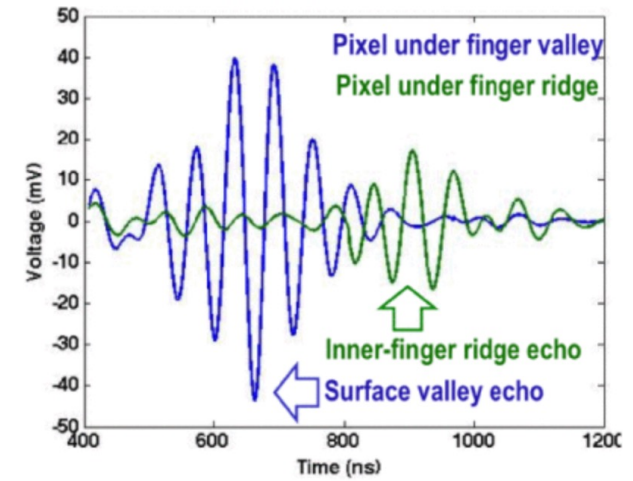
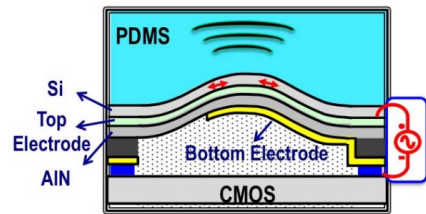
Roy et al, Microsystems and Nanoengineering 2023
 doi: 10.1038/s41378-023-00555-7.

EPFL Reading fingerprints



(a)

PMUT array, 20 MHz



H.-Y. Tang et al, 3-D Ultrasonic Fingerprint Sensor-on-a-Chip. *IEEE Journal of Solid-State Circuits* 51, 2522–2533 (2016).

Comparison of Capacitive vs. Ultrasonic Fingerprint Readers

Feature	Capacitive Sensor	Ultrasonic Sensor
Mechanism	Locally measure Capacitance	Emits ultrasonic waves that bounce off the fingerprint, capturing both surface and subsurface details.
Image Formation	Creates a map of capacitance to plot the fingerprint pattern.	Uses the time-of-flight and reflection strength of ultrasonic waves to create a 3D fingerprint image.
Feature	Capacitive Sensor	Ultrasonic Sensor
Detail Captured	Captures only the surface fingerprint ridges.	Captures both surface and subsurface details, making it more secure.
Spoof Resistance	Can be fooled by high-quality fingerprint molds (e.g., silicone replicas).	Harder to spoof since it captures subsurface details (e.g., blood flow and sweat pores).
Feature	Capacitive Sensor	Ultrasonic Sensor
Placement	Usually found on physical buttons (home button, back panel, or side).	Can be embedded under the display (e.g., in-screen fingerprint readers).
Complexity & Cost	Cheaper and easier to manufacture (~\$3-5 per sensor).	More expensive (~\$7-10 per sensor) due to advanced MEMS and ultrasonic tech.
Integration	Limited to physical locations (button, back, or side).	Works through glass, allowing for full-screen fingerprint scanning.

3. RF FILTERS

- For acoustic resonators, the mechanical wavelength in materials (μm scale at GHz) is several orders of magnitude smaller than the electromagnetic wavelength (cm scale at GHz in air)
 - Good fit for MEMS device
- RF Filters using piezos:
 - SAW: surface acoustic wave (< 2 GHz)
 - BAW: bulk acoustic wave

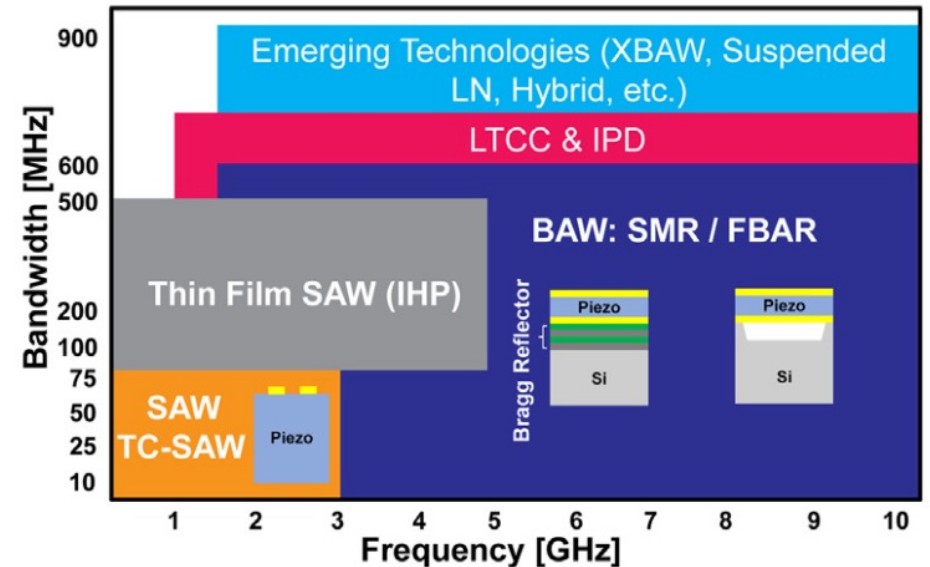


Fig. 2 | Advanced RF filters for wireless communications.

K. Yang, C. He, J. Fang, X. Cui, H. Sun, Y. Yang, C. Zuo, Advanced RF filters for wireless communications. *Chip* 2, 100058 (2023).

SAW: surface acoustic wave (< 2 GHz)

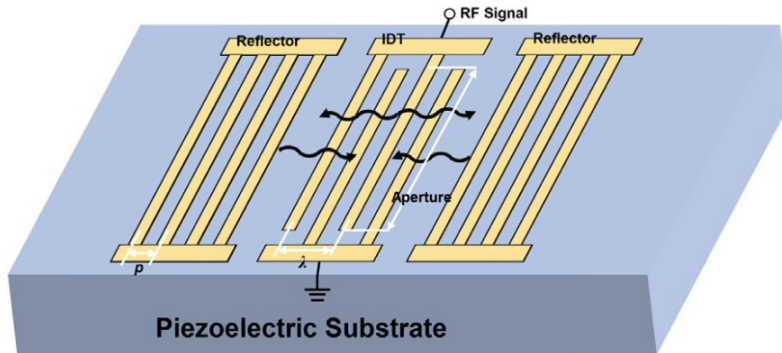
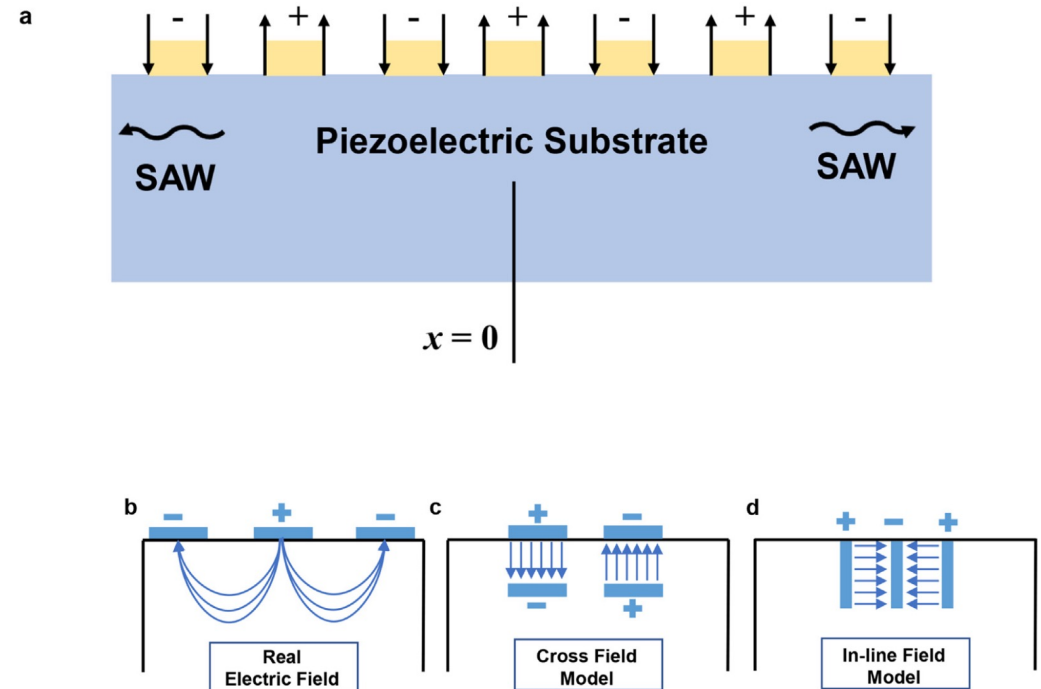


Fig. 3 | Typical structure of a SAW resonator.

$$f_0 = \frac{V_{SAW}}{\lambda}$$

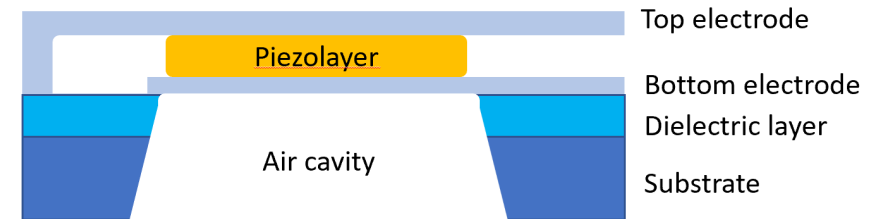
- λ is the wavelength, given by the periodicity of the IDT
- V_{SAW} : acoustic wave velocity.

$\lambda/4$ spacing between fingers defines frequency:
lithographically defined.



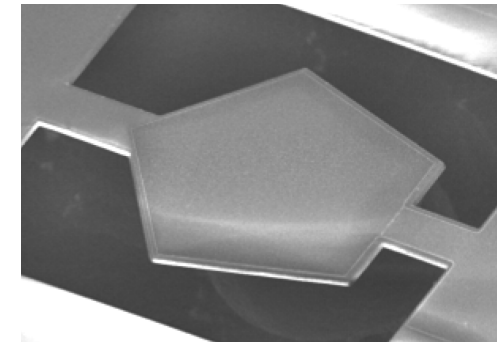
Piezoelectric resonator

- Use Piezoelectric effect to deform resonator... and read also out the motion
- Bulk Acoustic Resonator (BAR)
 - Acoustic waves constructive interaction
 - *Typical frequencies:* 100 MHz – 10 GHz
 - *Applications:* Filters, Oscillators
- *Advantages:*
 - High frequency, high Q
 - Easy electromechanical modelling
- *Disadvantages:*
 - Complicated design (to trim properties)
 - One frequency per wafer
- Piezo scales well down to small sizes: very effective to drive oscillators and filters
- Much lower voltage than capacitive MEMS. Direct drive with CMOS levels



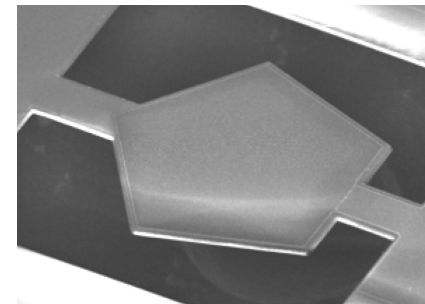
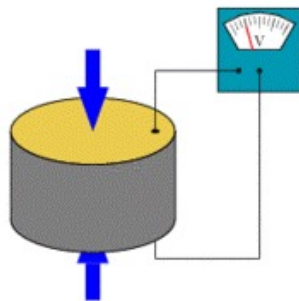
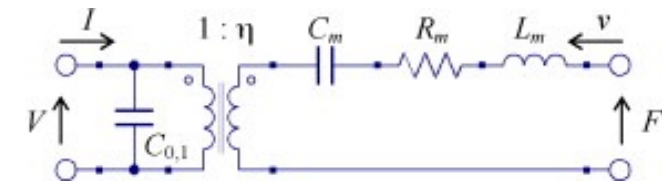
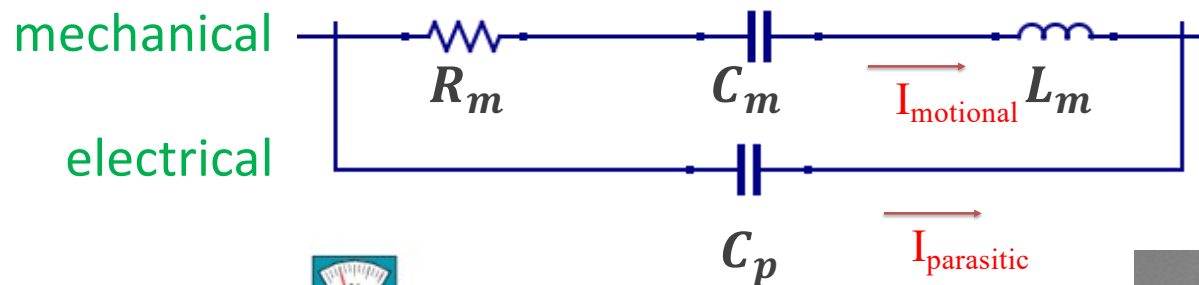
https://en.wikipedia.org/wiki/Thin-film_bulk_acoustic_resonator

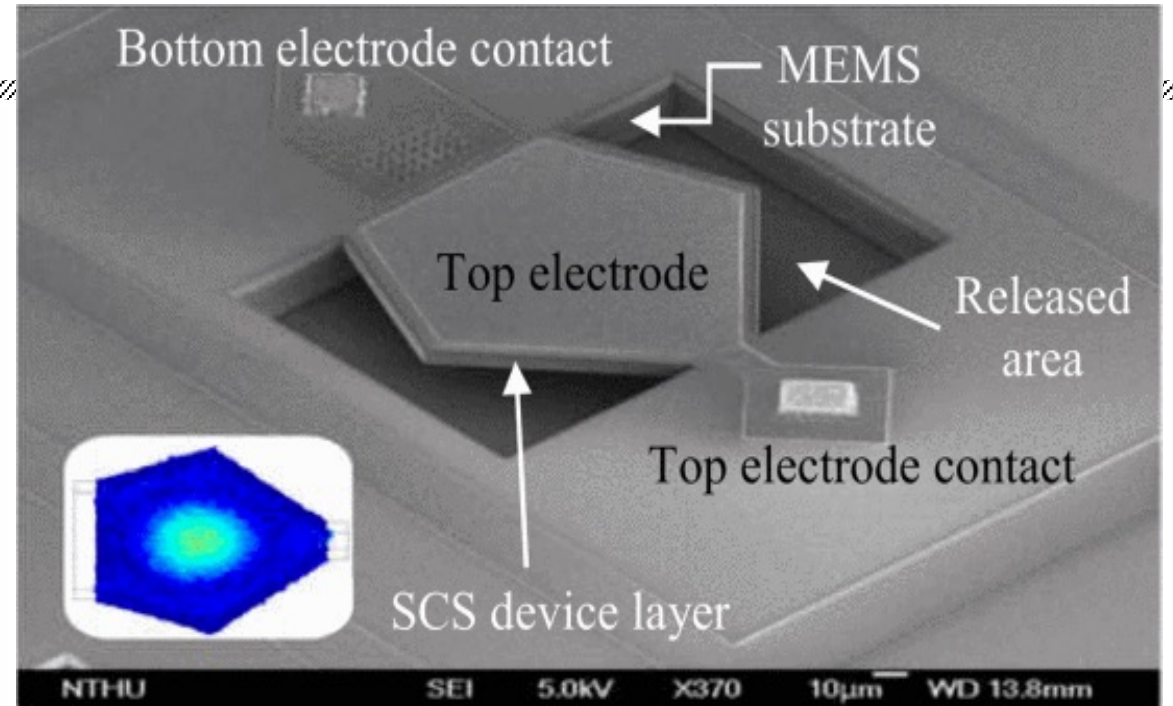
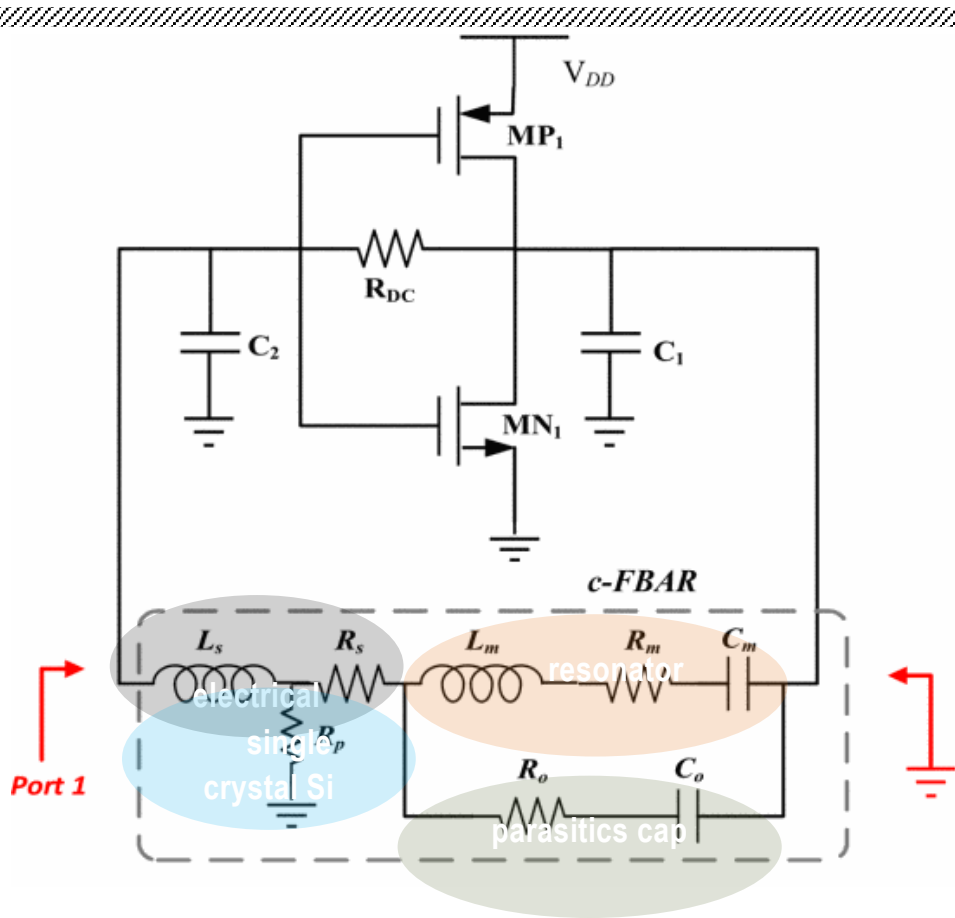
$$f \propto \sqrt{\frac{E n}{\rho t}}$$



Equivalent circuit

- When the mechanical resonator moves
 - Motion is amplified around the resonance frequency (Q factor)
 - This is a mechanical system, modelled as an LCR in series: *motional impedance*
- But charges will also move according to the physical capacitor
 - Thus another path for the charges is created – *parasitic capacitance*





$Q=2000$ at 3.25 GHz
 Single crystal silicon on Piezo

G. Pillai, et al, "3-GHz BAW composite resonators integrated with CMOS in a single-chip configuration," *2016 IEEE International Frequency Control Symposium (IFCS)*, New Orleans, LA, 2016, pp. 1-4. doi: 10.1109/FCS.2016.7563593

Bulk Acoustic Wave (BAW) filter

- BAW technology relies on thin film deposition of piezoelectric aluminum nitride (AlN) on a silicon (Si) substrate.
- The bulk acoustic wave travels and resonates in the vertical direction so that the thin film thickness determines the resonant frequency.
- Two ways of confining acoustic energy within the thin film piezoelectric layer:
 - MEMS process to suspend AlN plate at a certain gap from the Si substrate, = Film Bulk Acoustic Resonator (FBAR)
 - Or make Bragg reflector to reflect most of the acoustic energy back into the piezoelectric AlN plate, known as a Solidly Mounted Resonator (SMR)

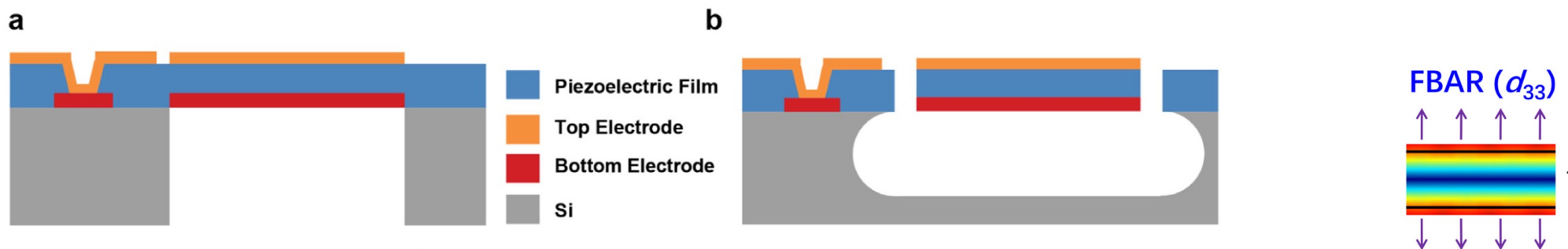
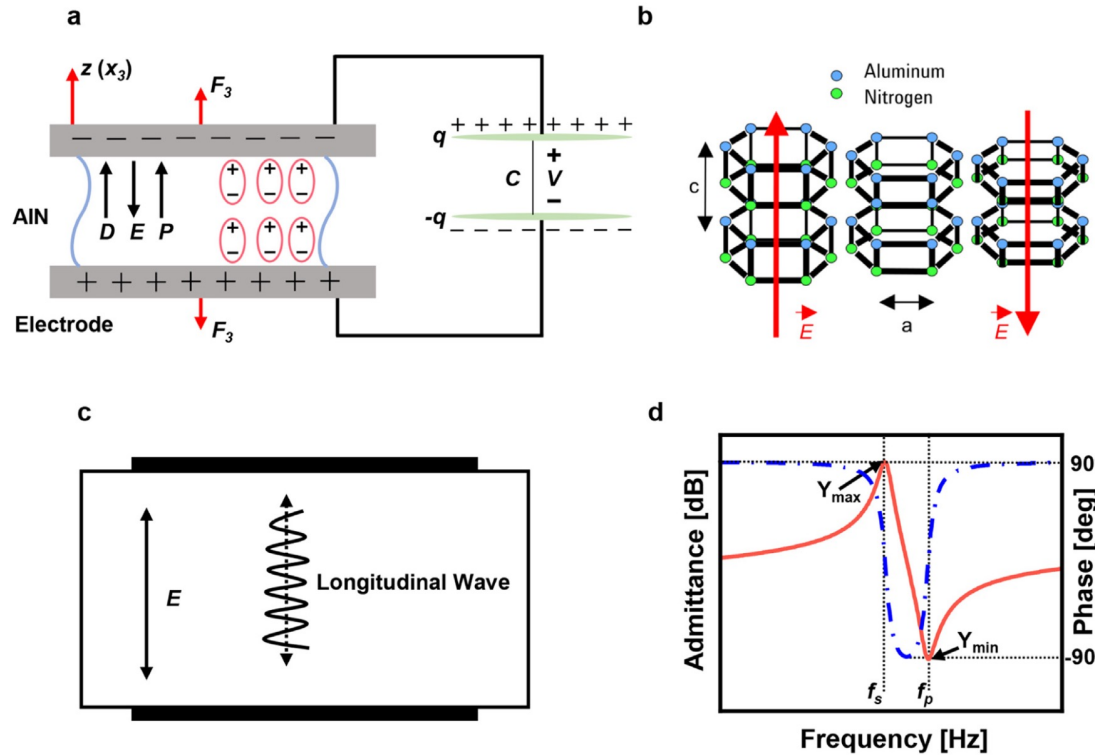


Fig. 9 | Cross-sectional drawing of FBAR device structures: **a**, membrane-type FBAR and **b**, airgap-type FBAR.

FBAR



- The resonant frequency of FBAR resonator is determined by the total thickness of the metal-piezo-metal thin film layer stack.
- Only one frequency can be obtained on the same wafer or chip with a fixed layer stack.

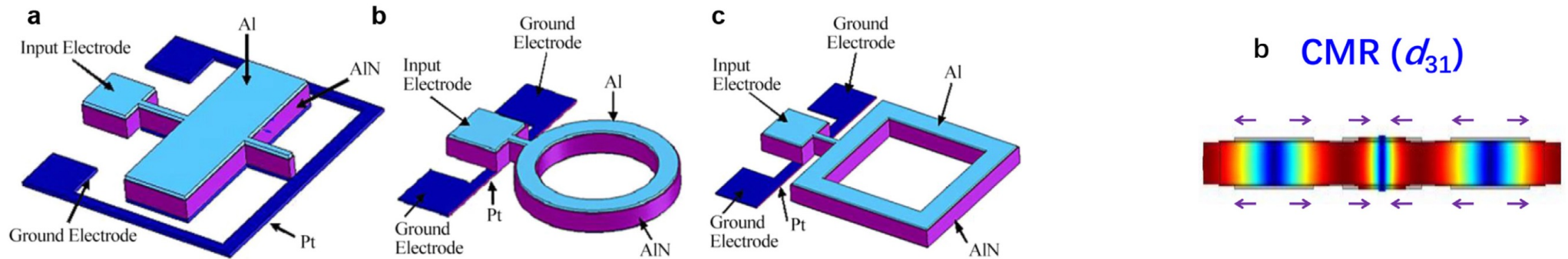
The longitudinal piezoelectric coefficient (d_{33}) is primarily the mechanism responsible for FBAR operation: vibration and voltage are both in the z direction

$$f_{res} = \frac{v}{t}$$

- v acoustic velocity in the piezoelectric material,
- t thickness of the piezoelectric layer.

Fig. 10 | a, Electric displacement and polarization in an FBAR resonator. Reprinted with permission from ref.¹⁰⁵. © 2008, Universitat Autònoma de Barcelona. b, Deformation of the crystal structure when electric field is applied in the c -axis. Reprinted with permission from ref.¹⁰⁶. © 2001, Avagotech. c, The direction of electrical field and acoustic wave propagation in an FBAR resonator. d, Typical frequency response of an FBAR resonator.

Contour-mode resonators (CMR): frequency is set by lateral dimensions



Three different designs of contour-mode resonators (CMR). a, Rectangular plate. b, Circular ring. c, Square-shaped ring. Reprinted with permission from [1], 2006, IEEE.

$$f_0 = \begin{cases} \frac{1}{2L} \sqrt{\frac{E_p}{\rho}} \\ \frac{1}{2W} \sqrt{\frac{E_p}{\rho}} \end{cases}$$

The transverse piezoelectric coefficient d_{31} is utilized to excite a mechanical vibration in the lateral (in plane) direction, while the voltage is applied in the z direction

- E_p equivalent Young's modulus of the metal-piezo-metal stack,
- ρ is the equivalent density
- L (or W) denotes the length (or width) of the rectangular plate depending on which of the length-extensional or width-extensional mode is used.

The resonant frequency of CMR can be easily set by lateral dimensions, so multi-frequency chips are possible